

On the simplicity of numbers

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Abstract

The recent measurements of reactor $\bar{\nu}_e$ disappearance and its interpretation in terms of the three light neutrino mixing angle θ_{13} by the DAYA BAY $\rightarrow \theta_{13} = \left(8.83^{+0.81}_{-0.88} \right)^\circ$ and RENO $\rightarrow \theta_{13} = \left(9.36^{+0.88}_{-0.96} \right)^\circ$ collaborations, gave rise to this treatise, upon the hypothetical substitution $\theta_{13} \rightarrow \vartheta_9 = 9^\circ$. The latter angle ($\vartheta_9 = 9^\circ$) is related to interesting algebraic properties of its periodic functions, which in turn have their origin in the discrete symmetry groups $S_5 = Z_2 \times A_5$ and A_5 , the point groups associated with the regular d = 3 'Platonic bodies': dodekahedron and ikosahedron. How these discrete groups may be related to dynamical symmetries of mass and mixing of light neutrino flavors is left open.

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1 Dodekahedron - Icosahedron and pentagon related planar symmetry

Algebraic identity for the angle $\vartheta_9 = 9^\circ$, related to the pentagon

$$\vartheta_9 = 9^\circ = 45^\circ / 5 : s_{13} = \sin(\vartheta_9) , c_{13} = \cos(\vartheta_9)$$

$$s_{13} = \sqrt{\frac{1}{2} \left(1 - \sqrt{1 - \left(\frac{\sqrt{5} - 1}{4} \right)^2} \right)}$$

$$c_{13} = \sqrt{\frac{1}{2} \left(1 + \sqrt{1 - \left(\frac{\sqrt{5} - 1}{4} \right)^2} \right)}$$
(1)

From eq. 1 we can infer the half-angle nature of the relation for $s_{13} = \sin(\vartheta_9)$

$$\text{let : } \sin(\varphi) = \frac{\sqrt{5} - 1}{4} = \frac{1}{1 + \sqrt{5}} \longrightarrow$$

$$\varphi = \vartheta_{18} = 2\vartheta_9 = 18^\circ$$
(2)

To clarify the context of the following considerations let me state : in no way they are meant to imply exact , and less so 'deep' principles, which would even in a clearly defined approximation, determine the exact values of the angles, pertinent to the mixing matrix of the three light neutrino flavors

$$\vartheta_{mn} ; (m \neq n) = 1, 2, 3$$
(3)

or periodic functions thereof .

This said, let me refer to the causes by recent events, that led to the following treatise on 'The simplicity of numbers' :

- 1) First was the announcement by the DAYA-BAY collaboration on the determination with remarkable statistical significance of the (anti-)neutrino mixing angle ϑ_{13} in ref. [2-2012] .
- 0) Really 1) was preceded by its announcement in correspondence by Zhi-zhong Xing, also a member of the DAYA-BAY collaboration, who, after exchange of e-mails, also with Werner Rodejohann, mentioned ref. [1-2009] .
- 2) Two papers deserve to be quoted in addition [3-2012] , [4-2012] .

2 The simplicity of numbers

From the Abstract of ref. [2-2012] , concerning the determination of ϑ_{13}
I quote : " A rate-only analysis finds

$$\sin^2 (2 \theta_{13}) = 0.092 \pm 0.016(stat.) \pm 0.005(syst.) \quad (4)$$

in a three-neutrino framework." It is consistent with the more recent results of the RENO collaboration , ref. [9-2012]

$$\sin^2 (2 \theta_{13}) = 0.103 \pm 0.013(stat.) \pm 0.011(syst.) \quad (5)$$

Using the notation, for experimental quantities

$$\begin{aligned} S13 &= \sin (\theta_{13}) ; S213 = \sin (2 \theta_{13}) \\ X &= (S13)^2 ; Y = (S213)^2 \end{aligned} \quad (6)$$

we have the functional relation

$$X (Y) = \frac{1}{2} (1 - \sqrt{1 - Y}) \quad (7)$$

Combining the errors in eq. 4 in quadrature we use

$$\sin^2 (2 \theta_{13}) = 0.092 \pm 0.017 (comb) \quad (8)$$

Thus we obtain

$$\begin{aligned} X (0.109 , 0.092 , 0.075) &= \begin{pmatrix} 0.0280360183234318 \\ 0.0235548300171362 \\ 0.0191153984582164 \end{pmatrix} \rightarrow \\ \theta_{13} &= \begin{pmatrix} 9.63898499442633^\circ \\ 8.8284100120672^\circ \\ 7.94708265426291^\circ \end{pmatrix} \sim (8.828^{+0.811}_{-0.881})^\circ \end{aligned} \quad (9)$$

The deviation $\Delta 9 = 9^\circ - \theta_{12}$ is less than 0.22 times the 1 sigma error .
Upon *choosing* $\vartheta_{13} \equiv \vartheta_9 = 9^\circ$, the 'simplicity of numbers' lies in the algebraic expression for $s13 \equiv \sin (\vartheta_9)$ given in eq. 1 .

3 The experimental determination of the neutrino mixing angles θ_{mn} ; $m < n = 1, 2, 3$ including theoretical analyses

In their own right , as witnessed by refs. [1-2009] - [6-2011], the recent weeks have seen a major step , experimentally, by the DAYA BAY results, reported in ref. [2-2012] , followed by the RENO collaboration in ref. [9-2012] , of strong evidence for the influence of θ_{13} , given in eqs. 8 and 9 :

$$X = (\sin(\theta_{13}))^2 = 0.0235548300171362 \begin{bmatrix} +0.0044811883062956 \\ -0.0044394315589198 \end{bmatrix}$$

$$X_9 = (\sin(9^\circ))^2 = 0.0244717418524232 \quad (10)$$

Going back in time from the communication by the DAYA BAY collaboration [2-2012] we turn towards the accelerator long baseline results by the T2K collaboration [6-2011] , which reports on limits inferred for the quantity $Y = (S_{213})^2 = (\sin(2\theta_{13}))^2$ defined in eqs. 6 and 8 comparing with the DAYA BAY and RENO [9-2012] results

$$Y = (\sin(2\theta_{13}))^2 = \begin{matrix} 0.092 \pm 0.016(stat.) \pm 0.005(syst.) ; \text{DAYA} \\ \text{BAY} \\ 0.103 \pm 0.013(stat.) \pm 0.011(syst.) ; \text{RENO} \end{matrix}$$

$$Y_{18} = (\sin(18^\circ))^2 = (3 - \sqrt{5}) / 8 = 0.0954915028125263 \quad (11)$$

The 90 % confidence limits for Y as reported by the T2K collaboration are derived for $\delta_{CP} = 0$ and separately for a) normal and b) inverted hierarchy of light neutrino masses

$$\begin{aligned} \text{a) } & 0.03 < (\sin(2\theta_{13}))^2 < 0.28 \quad \text{for normal hierarchy} \\ & \delta_{CP} = 0 \quad ; \quad \text{T2K} \\ \text{b) } & 0.04 < (\sin(2\theta_{13}))^2 < 0.34 \quad \text{for inverted hierarchy} \end{aligned} \quad (12)$$

We turn to the analysis of ref. [5-2011] , which we first compare with eq. 10 using the values corresponding to newly corrected reactor fluxes at 1σ

$$X = (\sin(\theta_{13}))^2 = 0.0235548300171362 \begin{bmatrix} +0.0044811883062956 \\ -0.0044394315589198 \end{bmatrix}$$

$$\text{DAYA BAY}$$

$$X = (\sin(\theta_{13}))^2 = 0.025 \pm 0.007 ; \text{ ref. [5-2011]}$$

$$X_9 = (\sin(9^\circ))^2 = 0.0244717418524232 \quad (13)$$

Next we consider the 3σ range of $Z = (\sin(\theta_{12}))^2$ in ref. [5-2011]

$$0.265 < Z = (\sin(\theta_{12}))^2 < 0.364 \quad ; \quad \text{ref. [5-2011]} \quad (14)$$

$$Z_{36} = (\sin(36^\circ))^2 = (5 - \sqrt{5}) / 8 = 0.345491502812526$$

and similarly $ZZ = (\sin(\theta_{23}))^2$

$$0.34 < ZZ = (\sin(\theta_{23}))^2 < 0.64 \quad ; \quad \text{ref. [5-2011]} \quad (15)$$

$$ZZ_{45} = (\sin(45^\circ))^2 = 0.5$$

Let me stress once more, that there is no relation – known to me – between the number 5 and associated angles within the ‘simplicity of numbers’

$$\vartheta_{13} \leftrightarrow 9^\circ, \quad \vartheta_{12} \leftrightarrow 36^\circ, \quad \vartheta_{23} \leftrightarrow 45^\circ \quad (16)$$

with the pentagon, dekahedron as regular 2 dimensional structures, nor with the dual pair dodekahedron - ikosahedron, as regular 3 dimensional ones, which could be associated with a physically meaningful symmetry, limiting or absolute.

Any such physical symmetry should also account for the breaking pattern of the unifying gauge group spin 10 or D_5 , and then involves roots and weights, i.e. polyhedra in 5 dimensions (= the rank of D_5). In particular such symmetry relation should include ‘crystalline axes’ thereof, along which this group is broken as discussed in refs. [7-2005], [8-2008].

The decomposition of one family of the D_5 16 dimensional spinor representation and its decomposition along the subgroup axes corresponding to $D_5 \supset SU5 \times U1$ can serve here as a guideline

$$[16] = \{1, 5\} + \{10, 1\} + \{\bar{5}, -3\} \quad (17)$$

For completeness let me add two recent references on neutrino flavor oscillations : ref. [10-2012] on CP-phases and the discrete group T_7 and ref. [11-2012] a review on leptonic CP violation.

A1 - Constructing the dodekahedron

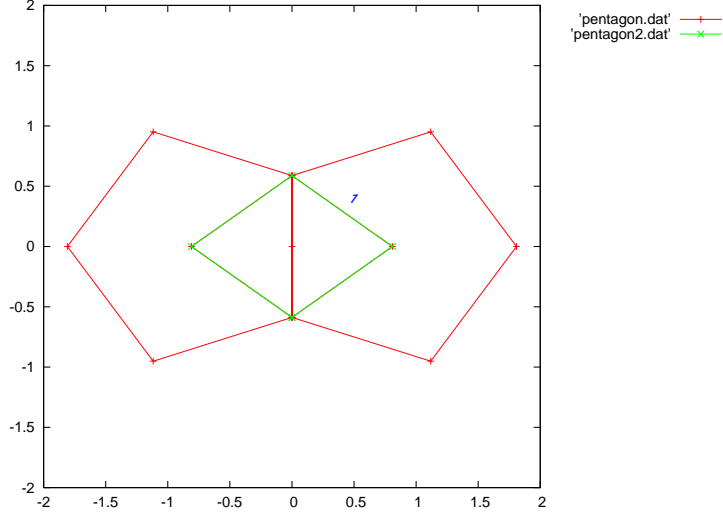


Fig. 1 : Two pentagons in one plane
 $(x \rightarrow), (y \uparrow), (z \odot)$

We consider the x-axis to be parallel to the abscissa, the y-axis parallel to the ordinate in Fig. 1 and the z-axis completing a right oriented orthogonal basis, pointing out of the x-y plane of the figure. The distance from the center of the pentagons to its points is taken as unity, as shown in Fig. 1 . The two pentagon points lowest along the negative y-axis thus have the cartesian coordinates (eq. 2)

$$\begin{aligned} \text{PR}(L) : & \left(\pm \cos(18^\circ) L, -\left(\frac{1}{2} + \sin(18^\circ)\right) L, 0 \right) \\ \cos(18^\circ) = & \sqrt{\frac{5 + \sqrt{5}}{8}}, \quad \sin(18^\circ) = \frac{\sqrt{5} - 1}{4} \\ L = 2 \sin(2\varphi) = & \sqrt{\frac{5 - \sqrt{5}}{2}}; \quad 2\varphi = 36^\circ \end{aligned} \quad (18)$$

We rotate the right side pentagon around the y-axis by the angle ψ in order to form an angle of 108° instead of 144° between the nearest edge of the left side pentagon and the rotated nearest edge of the right side one

$$R_y(\psi) \begin{pmatrix} z \\ x \end{pmatrix} = \begin{pmatrix} \cos(\psi) z - \sin(\psi) x \\ \sin(\psi) z + \cos(\psi) x \end{pmatrix} = \begin{pmatrix} -\sin(\psi) x \\ \cos(\psi) x \end{pmatrix} \quad (19)$$

The so rotated points $\text{PR}' = R_y(\psi) \text{PL}$ and PL display the coordinates

$$\begin{pmatrix} \text{PR}' \\ \text{PL} \end{pmatrix} = \begin{pmatrix} +\cos(\psi) \cos(\varphi) & , & -\sin(\varphi) & , & -\sin(\psi) \cos(\varphi) \\ -\cos(\varphi) & , & -\sin(\varphi) & , & 0 \end{pmatrix}$$

$\varphi = 18^\circ$

(20)

Thus we determine the sought angle between the two vectors displayed in eq. 20 from the relation

$$\begin{aligned} \cos(\text{PR}', \text{PL}) &= -(\cos(\psi) \cos^2(\varphi) - \sin^2(\varphi)) \\ &= -\sin(\varphi) = \cos(108^\circ) \end{aligned}$$
(21)

which yields

$$\begin{aligned} \cos(\psi) &= \tan^2(\varphi) + \frac{\sin(\varphi)}{\cos^2(\varphi)} = \frac{1}{\frac{\sqrt{5}}{2}} \\ \sin(\psi) &= \frac{2}{\sqrt{5}} \end{aligned}$$
(22)

Eq. 22 establishes the algebraic simplicity of the angle ψ in the sense of 'simplicity of numbers'.

ψ itself is *not* a multiple of 9° however

$$\begin{aligned} \psi &= \arctg(2) = 63.434948822922^\circ \\ \alpha &= 180^\circ - \psi = 116.565051177078^\circ \end{aligned}$$
(23)

The angle between two planes of the dodekahedron, which have one edge in common, called α or dihedral angle, is defined as $\alpha = 180^\circ - \psi$ in eq. 23.

Centering the faces

Having chosen the distance from the center of a pentagon to its points of length unity as in Fig. 1, we turn to the center points of the 12 pentagons forming the dodekahedron.

Let CL be the center point of the unrotated left pentagon, and CR' the center point of the rotated right pentagon in Fig. 1. The associated two pentagons shall be denoted $\text{penta}(\text{L})$ and $\text{penta}(\text{R}')$ respectively.

They have coordinates, centered as in Fig. 1, using eq. 19

$$\begin{aligned} \text{CL} &: (-\cos(2\varphi), 0, 0) \\ \text{CR}' &: (\cos(\psi) \cos(2\varphi), 0, -\sin(\psi) \cos(2\varphi)) \end{aligned}$$
(24)

The components of the outer normal to $\text{penta}(\text{R}')$, denoted $(\vec{n})'$, are

$$(\vec{n})' = (\sin(\psi), 0, \cos(\psi))$$
(25)

Thus we find the center of the dodekahedron intersecting the two straight lines orthogonal to the faces , which lie in the $y = 0$ plane.

This gives rise to the equation by eq. 20

$$\begin{aligned}
- \cos (2 \varphi) &= - \lambda \sin (\psi) + \cos (\psi) \cos (2 \varphi) \rightarrow \\
\lambda &= \cos (2 \varphi) \frac{1 + \cos (\psi)}{\sin (\psi)} \\
&= 2 \left(\frac{1 + \sqrt{5}}{4} \right)^2 = 1.30901699437495
\end{aligned} \tag{26}$$

λ as defined in eq. 26 is the distance of any center point of the (12) pentagons from the center of the dodekahedron, and thus is the radius of the largest sphere inscribed within it, touching the faces from inside at the twelve center points of pentagons.

These 12 pentagon center points form 12 base points of an ikosahedron.

We now translate the d=3 coordinates used for CL in eq. 24 such that the center of the dodekahedron (to be) is at the origin. Thus we introduce new coordinates $\vec{x}_1 = \vec{x} + \vec{T}$ according to the transformation

$$\begin{aligned}
\vec{x}_1 &= (x1 , y1 , z1) ; \vec{x} = (x , y , z) \\
\vec{T} &= (\cos (2 \varphi) , 0 , \lambda) \\
&= (0.809016994374947 , 0 , 1.30901699437495)
\end{aligned} \tag{27}$$

Next we go one step back to the 3d coordinates of \vec{x} and determine the coordinates of the center point called CR' in eq. 24 , repeated below

$$\begin{aligned}
\text{CL} &: (- \cos (2 \varphi) , 0 , 0) \\
\text{CR}' &: (\cos (\psi) \cos (2 \varphi) , 0 , - \sin (\psi) \cos (2 \varphi))
\end{aligned} \tag{28}$$

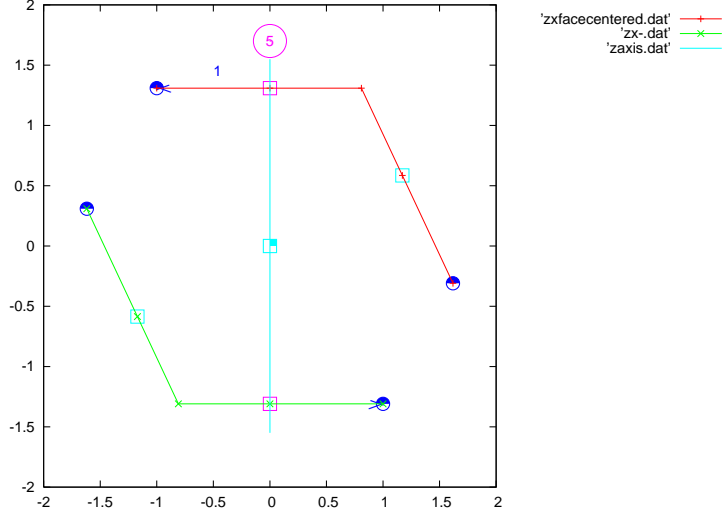


Fig. 2 : The section $y = 0$ as xz -plane
 $(x \rightarrow), (y \otimes), (z \uparrow)$

The coordinates x, y, z used in Fig. 2 represent the body centered system, pertaining to the translated vectors $\vec{x}_1 = \vec{x} + \vec{T}$, as defined in eq. 27. The suffix $_1$ is not explicitly indicated for compactness of notation. Further in Fig. 2 the z -axis is drawn in cyan color. It is identical to the fivefold axis and this is marked by the encircled number 5. The four half filled blue circles denote 4 points of the dodekahedron, from which the five rotations around the z -axis generate the 20 points or corners of the dodekahedron.

The four empty quadrangles denote those center points of the 4 pentagons, which lye in the $y = 0$ plane drawn in Fig. 2. Upon performing the five symmetry rotations around the z -axis the two quadrangle points, lying on it, remain invariant, whereas the other two generate the 10 points, which complete the twelve center points of the associated pentagons. These 12 points form basis corners of an ikosahedron.

Finally the quarter filled quadrangle in Fig. 2 marks the center of the dodekahodron as well as of the associated ikosahedron.

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Version 1	31.03.2012
Results of the RENO collaboration, ref. [9-2012], added	04.04.2012
Version 2	07.04.2012
Version 3	15.04.2012

Additional material



Fig B1 : Skeleton of a an elephant or mammoth
[http://www.anatomy.mvm.ed.ac.uk/museum/includes
/img.php?crop=full/&img=../upload/exhibits/elephant.jpg](http://www.anatomy.mvm.ed.ac.uk/museum/includes/img.php?crop=full/&img=../upload/exhibits/elephant.jpg)